Development

## DECIMAL PLACES AND SIGNIFICANT FIGURES

## DECIMAL PLACES

Sometimes you are required to give a shorter answer than the one which you have worked out.

## Example 1

3.68472 is your answer, but you are asked to give the answer
"correct to 2 decimal places"
The answer will then be:

### 3.68 (correct to 2 decimal places).

In order to arrive at this answer, you must look at the third number after the decimal point, even though, in your answer, there will be only two figures after the point.

If the third number is

## $0,1,2,3$ or 4

then the second number will remain the same (as in the example given above).
If the third number is

## 5, 6, 7, 8 or 9

then the second number is increased by 1.

## Example 2

Give 3.68872 correct to 2 decimal places.
Look at the third figure after the point, it is 8 , and because it is in the group $5,6,7,8,9$ then the second figure is increased by 1 so the answer is:-

### 3.69 correct to 2 decimal places.

You may be asked to give a number correct to any number of decimal places. Remember if you are asked to give a number correct to 1 decimal place, you look at the $2^{\text {nd }}$ figure.

1 decimal place, you look at the $2^{\text {nd }}$ figure.
2 decimal places, you look at the $3^{\text {rd }}$ figure.
3 decimal places, you look at the $4^{\text {th }}$ figure.
4 decimal places, you look at the $5^{\text {th }}$ figure, etc.

## Example 3

12,051 correct to 1 decimal place $=12.1$
(The $2^{\text {nd }}$ figure is 5 , so increase 0 by 1 , giving 1.)
12,051 correct to 2 decimal places $=12.05$
(The $3^{\text {rd }}$ figure is 1 , so the 5 remains the same.)

## Example 4

0.675 correct to 1 decimal place
(The $2^{\text {nd }}$ figure is 7 , so the 6 is increased by 1 )
0.675 correct to 2 decimal places $=0.68$
(The $3^{\text {rd }}$ figure is 5 , so the 7 is increased to 8 ).

## Example 5

0.0517 correct to 1 decimal place $=0.1$
0.0517 correct to 2 decimal places $=0.05$
0.0517 correct to 3 decimal places $=0.052$

## Example 6

16.97 correct to 1 decimal place $=17.0$

## Exercise 1

Write down why these answers are true

1. 16.00726 correct to 1 d.p. $=16.0$
2. 16.00726 correct to 2 d.p. $=16.01$
3. 16.00726 correct to $3 \mathrm{~d} . \mathrm{p} .=16.007$
4. $\quad 16.00726$ correct to 4 d.p. $=16.0073$

## Exercise 2

Write each of these numbers correct to:
a) 1 decimal place
b) 2 decimal places

1. 63.147
2. 4.093
3. 5.088
4. 1.008
5. 3.927

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## SIGNIFICANT FIGURES

The amount of approximation required in a number may be described in another way by saying how many significant figures are required.

To find how many significant figures a number contains count all figures in the number except zeros at the beginning or end of the number.

## Example 1

3625.4 has 5 significant figures
0.0023 has 2 significant figures

360 has 2 significant figures
You may be asked to approximate a given number correct to so many significant figures. This process is carried out in exactly the same way as for decimal places. When examining the first figure we are discarding use the groups $0-4$ and 5-9.

## Example 2

8.619 has 4 significant figures, most important is the figure $\mathbf{8}$ in the units column.
$8.619=9$ correct to 1 significant figure
$8.619=8.6$ correct to 2 significant figures
$8.619=8.62$ correct to 3 significant figures

## Example 3

23.61 has 4 significant figures, the figure ' 2 ' is the most important.
23.61 = 20 correct to 1 significant figure (not just 2; we need to put in a zero to show that we mean twenty).
$23.61=24$ correct to 2 significant figures
$23.61=23.6$ correct to 3 significant figures
NOTE that the 2 (in 23) means two tens.

## Example 4

127.9 has 4 significant figures
$127.9=100$ correct to 1 significant figure (needs 2 zeros as "spacers" to make the 1 mean 1 hundred).
$127.9=130$ correct to 2 significant figures
$127.9=128$ correct to 3 significant figures

## Example 5

4309 has 4 significant figures. Notice that we count the zero between 3 and 9 as it is in the middle of the number and is surrounded by significant figures
$4309=4000$ correct to 1 significant figure
$4309=4300$ correct to 2 significant figures
$4309=4310$ correct to 3 significant figures
Here, the 4 in 4309 means four thousand.

## Example 6

0.273 has 3 significant figures
$0.273=0.3$ correct to 1 significant figure
$0.273=0.27$ correct to 2 significant figures

## Example 7

0.0915 has 3 significant figures
$0.0915=0.09$ correct to 1 significant figure
$0.0915=0.092$ correct to 2 significant figures

## Exercise 3

a) correct to 1 significant figure
b) correct to 2 significant figures
c) correct to 3 significant figures

1. 14.7541
2. $\quad 165.5$
3. 37654.21
4. 0.4915
5. 0.007086

## APPROXIMATION OR ESTIMATIONS

Often we do not need to give an exact answer and are asked to give a rough idea of an answer.

## Example 1

If you spend 53 p you may say that you have spent 'about 50p'.
If you spend 57 p you may say that you have spent approximately 60 p.

If you spend 55 p we round this up to 60p.
So if you spend $50,51,52,53$ or 54 , we say it is approximately 50 p and if you spend $55,56,57,58$ or 59 , we would say that it is approximately 60 p.

A question may ask for the answer 'to the nearest 100' 'or 10' as in the following:

## Example 2

$304 \times 17.304$ is approximately 300 and 17 is approximately 20.
So $300 \times 20$ is 6000, so the answer to $304 \times 17$ will be in the region of 6000 .

## Example 3

If you were to buy a car costing $£ 3750$, you would say that this was $£ 4000$ to the nearest $£ 1000$ or that it was $£ 3800$ to the nearest $£ 100$.

## Example 4

If you and three friends went out for a meal and the bill came to $£ 12.47$ you would each pay approximately $£ 3.00$.

## Example 5

Estimate $334 \times 18$ to the nearest 100. 334 is about 300 and 18 is about 20.

So the answer will be in the region of 6000.
Estimate the same sum to the nearest 10.

334 is roughly 330 and 18 is roughly 20.
So the answer will be in the region of 6600.

## Example 6

$33 \times 6=198$

You can use this information to work out examples using the same numbers but having different values:
e.g. $3.3 \times 6=19.8$
$33 \times 0.6=19.8$
$3.3 \times 0.6=1.98$
$3.3 \times 0.06=0.198$
$0.33 \times 0.6=0.198$

## Exercise 4

Estimate the following:

1. $34 \times 17$ to the nearest 100 by rounding off the numbers to nearest 10 .
2. $233 \times 117$ to the nearest 100 by rounding off the numbers to nearest 10 .
3. $346 \times 274$ to the nearest 100 by rounding off the numbers to nearest 10 .
4. $\frac{346}{47}$
5. $\frac{3.46}{47}$

## Exercise 5

If $13 \times 4=52$ give the answers to the following:

1. $1.3 \times 4$
2. $1.3 \times 0.4$
3. $0.13 \times 0.4$
4. $0.13 \times 4$
5. $0.13 \times 0.04$

## ANSWERS

## Exercise 1

| 16.00726 | correct to 1 d.p. $=16.0$ <br> because $2^{\text {nd }}$ figure after d.p. $=0$ <br> 16.00726 |
| :--- | :--- |
| correct to 2 d.p. $=16.01$  <br> because $3^{\text {rd }}$ figure after d.p. $=7$ |  |
| 16.00726 | correct to 3 d.p $=16.007$ <br> because $4^{\text {th }}$ figure after d.p. $=2$ |
| 16.00726 | correct to 4 d.p. $=16.0073$ <br> because $5^{\text {th }}$ figure after d.p. $=6$ |

## Decimal Places

## Exercise 2

1. 63.1 (to 1 d.p.) 63.15 (to 2 d.p.)
2. 4.1 (to 1 d.p.) 4.09 (to 2 d.p)
3. 5.1 (to 1 d.p.) 5.09 (to 2 d.p.)
4. 1.0 (to 1 d.p.) 1.01 (to 2 d.p.)
5. 3.9 (to 1 d.p.) 3.9. (to 2 d.p.)

## Significant Figures

## Exercise 3

|  | $\mathbf{1}$ sig. figure | $\mathbf{2}$ sig. figure | $\mathbf{3}$ sig. figure |
| :--- | :--- | :--- | :--- |
| 1. | 10 | 15 | 14.8 |
| 2. | 200 | 170 | 166 |
| 3. | 40000 | 38000 | 37700 |
| 4. | 0.5 | 0.49 | 0.492 |
| 5. | 0.007 | 0.0071 | 0.00709 |

## Exercise 4

1. 600
2. 27600
3. 94500
4. 7
5. 0.07

## Exercise 5

1. 5.2
2. 0.52
3. 0.052
4. 0.52
5. 0.0052

## Special Case

Applies when there is only one figure after the number of decimal places or significant figures being considered and that figure is a 5.

Rule: Round to the nearest even number
Example 1 Corrected to 1 decimal place
65.65 becomes 65.6 (i.e. rounded down)
65.75 becomes 65.8 (i.e. rounded up)

Example 2 Corrected to 3 significant figures
12350 becomes 12400
12850 becomes 12800
Example 3 Corrected to the nearest whole number
31.5 becomes 32
32.5 becomes 32

Example 4 Corrected to the nearest thousand
7500 becomes 8000
8500 becomes 8000
This is a "British Standard" and the reason for this is to reduce errors. If all the numbers ending in five were rounded up, then all the errors would be in the same direction. When some are rounded up and some rounded down the total result of the errors is compensated.

