LINEAR EQUATIONS

Here are some examples of linear equations:

1. $2a = 8$
2. $a + 2 = 4$
3. $3a + 6 = a + 10$
4. $3(a + 2) = 9$
5. $4a - 11 = 5a + 19$
6. $2(a - 1) - 4(3a = 6) = 10$
7. $\dfrac{b}{2} = 3$
8. $\dfrac{3a}{4} = 6$

In all these examples, there are letters and numbers on both sides of the equals sign and the letters have no powers higher than 1. (i.e. there are no a^2 or a^3 or b^2 or b^3 terms)

Your answer must have a letter, which must be **POSITIVE**, on one side of the = sign, and a number on the other side. For this to happen you must alter the equation to solve what the letter is.

It does not matter which side of the equals sign the letter is!

To solve simple equations you must follow a set of rules:

1. Remove any brackets by multiplying them out.

   e.g. 4 (above) $3(a + 2) = 9$
   becomes $3a + 6 = 9$

2. Put all the terms containing letters on one side and numbers on the other side.

   e.g. $3a + 6 = 9$
   $3a = 9 - 6$

   NB There are two ways of 'moving' numbers and letters.

a) When a term 'goes over' the = sign to the opposite side, the sign is changed:
   + becomes –
   - becomes +
   x becomes ÷
   ÷ becomes x

   e.g. $3a + 6 = 9$
   becomes $3a = 9 - 6$
   $3a = 3$
   $3 \times a = 3$
   $a = \dfrac{3}{3} = +1$

   Here the + 6 has become – 6, as it 'goes over' the equals sign and the x 3 becomes ÷ 3.

OR

b) You may balance an equation by doing the same thing to both sides.

$$3a + 6 = 9$$
$$3a + 6 - 6 = +9 - 6$$
$$3a = +3$$
$$+3 \times +1a = +3$$
$$+1a = \frac{+3}{+3} = +1$$

What you have done here is to **SUBTRACT** 6 from the left side, so, to balance the equation you must subtract 6 from the right side and you have **DIVIDED BOTH** sides by +3.

So finishing off example 2b

$$3a = 9 - 6$$
$$3a = 3$$
$$3 \times 1a = 3$$
$$1a = \frac{3}{3}$$
$$a = 1$$

CHECK

$$3a + 6 = 9$$
$$a = 1$$
3a becomes (3 x 1)
$$3 + 6 = 9 \textbf{ CORRECT}$$

Working through the example given at the beginning

1.
$$2a = 8$$
$$2 \times 1a = 8$$
$$1a = \frac{8}{2} = 4$$
$$a = 4$$

CHECK

$$2a = 8$$
$$2 \times 4 = 8 \quad \textbf{CORRECT}$$

2.
$$a + 2 = 4$$
$$a = 4 - 2$$
$$a = 2$$

CHECK

$$2 + 2 = 4 \textbf{ CORRECT}$$

3. 　　　　　　　$3a + 6 = a + 10$

    becomes $3 \times a + 6 = a + 10$

    REMEMBER from directed numbers that $3 = +3$ and $a = +1a$

    so, $3a - 1a = 10 - 6$

    　　　　$2a = 4$

    　　　　$1a = \dfrac{4}{2}$

    　　　　$a = 2$　　Check this answer yourself.

4. Already solved

5. There are two ways of solving this equation:

    First Way　　$4a - 11 = 5a + 19$

    　　　　　　　$+4a - 5a = 19 + 11$

    　　　　　　　　$-1a = +30$

    'a' must be positive, so **ALL** terms **MUST** be **MULTIPLIED BY -1** therefore $a = -30$.

    Second Way　　$4a - 11 = 5a + 19$

    　　　　　　　$-11 - 19 = 5a - 4a$

    　　　　　　　　$-30 = 1a$

    　　　　　　　　$-30 = a$

    Either way is correct

6. **First Way**　$2(a - 1) - 4(3a + 6) = 10$

    Do not forget – the minus sign outside a bracket changes the signs inside the brackets when the brackets are removed so this becomes

    　　　　$2a - 2 - 12a - 24 = 10$

    gather together all **LIKE** terms

    　　　　　$-10a - 26 = 10$

    　　　　　　$-10a = 10 + 26$

    　　　　　　$-10a = +36$

    multiply all terms by -1 to give:

    　　　　　　$+10a = -36$

    　　　　　　$+1a = -\dfrac{36}{+10}$

    　　　　　　$a = -3.6$

    Second Way

    　　　　　$-10a - 26 = 10$

    　　　　$-26 - 10 = +10a$

    　　　　　$-36 = +10a$

    　　　　　$-\dfrac{36}{+10} = +1a$

    　　　　　$-3.6 = a$

7. $$\frac{b}{2} = 3$$

$\div 2$ 'goes over' the equal sign to become x 2, to give

b = 3 x 2

b = 6

You have in fact, multiplied both sides by 2 to balance the equation as follows:

$$\frac{b}{2} = 3$$

MULTIPLY both sides by 2

$$\frac{2}{1} \times \frac{b}{2} = 3 \times 2 \qquad \text{cancel to give}$$

b = 6

8. $$\frac{3a}{4} = 6$$

MULTIPLY both sides by the denominator 4, to give:

$$\frac{4}{1} \times \frac{3a}{4} = 6 \times 4$$

CANCEL 4's to give

3a = 24

3 x a = 24

a = $\frac{24}{3}$

a = 8

Choose the method you like best and do these:

1.   3a = 12

2.   c + 3 = 7

3.   b - 2 = 5

4. $\frac{b}{3} = 4$

5.   2a + 5 = 9

6.   5a - 3 = 22

7.   3a + 11 = 35 − a

8.   4(g + 1)= 8

9.   3(b − 1) − 2(3b − 2) = 4

10. 4(a - 5) = 7 − 5(3 − 2a)

ANSWERS

1. $a = 4$
2. $c = 4$
3. $b = 7$
4. $b = 12$
5. $a = 2$
6. $a = 5$
7. $a = 6$
8. $g = 1$
9. $b = -1$
10. $a = -2$