## TRANSPOSITION OF FORMULAE

This is also known as Changing the Subject of the Formula

$$
Y=m \chi+c \quad \text { In this case, } \mathrm{y} \text { is the subject of the formula. }
$$

There are different ways of solving these, because there are so many different formulae.
Read through the following rules. They will help you to identify various types of formulae, and so help you with the problems you meet.

## RULES

a) Remove all fractions, brackets and roots.
b) Put ALL the terms containing the new subject on one side of the formula, and all the others on the other side.
c) Simplify both sides of this new formula. If the new subject still appears in more than one term(s), then treat it as a common factor. Then divide both sides of the formula by the terms in front of the new subject.

That is a lot to remember at one go! Do not worry! Keep referring back to these, as you work through the examples. Practice makes perfect!

## Example 1

$$
\mathrm{c}=\pi \mathrm{d}
$$

c is the subject of the formula
The question is "Make $d$ the subject of the formula". This means that $d$ must be put on its own on one side of the equals sign, and the other terms must be on the other side.

To do this, first DIVIDE both sides by $\pi$.

$$
\begin{aligned}
& \mathrm{c}=\pi \mathrm{d} \\
& \frac{\mathrm{c}}{\pi}=\frac{\pi \mathrm{d}}{\pi} \quad \text { cancel } \pi \text { to give } \quad \frac{\mathrm{c}}{\pi}=\mathrm{d}
\end{aligned}
$$

## Example 2

$$
\mathrm{V}=\pi \mathrm{r}^{2} \mathrm{~h}
$$

Make $h$ the subject of the formula. To obtain $h$ by itself, divide both sides by $\pi r^{2}$.

$$
\frac{\mathrm{V}}{\pi r^{2}}=\frac{\pi r^{2} h}{\pi r^{2}} \text { to give }=\frac{\mathrm{V}}{\pi r^{2}}=\mathrm{h}
$$

## Example 3

$$
\mathrm{a}=\frac{\mathrm{b}}{\mathrm{c}}
$$

Make $b$ the subject of the formula.
First multiply both sides by the denominator, c , to obtain b on its own:-

$$
a \times c=\underline{b \times c} \text { cancel } c \text { to give } a c=b
$$

## Example 4

$$
a=\frac{b}{c}
$$

This is the same formula as in the last example, but this time you must make c the subject of the formula.
Multiply both sides by the denominator, c , giving:-

$$
\mathrm{a} \times \mathrm{c}=\frac{\mathrm{b} \times \mathrm{c}}{\mathrm{c}} \text { cancel giving } \mathrm{ac}=\mathrm{b}
$$

cancel c giving $\mathrm{ac}=\mathrm{b}$
Now divide both sides by a, to give c on its own.

$$
c=\frac{b}{a}
$$

## Example 5

$$
y=m \chi+c
$$

Make c the subject of the formula.

$$
\mathrm{y}-\mathrm{m} \chi=\mathrm{c} \text { taking } \mathrm{mc} \text { from both sides }
$$

## Example 6

Using the same formula, make $\chi$ the subject of the formula

$$
\begin{aligned}
& y-c=m \chi \text { taking } c \text { from both sides } \\
& \frac{\mathrm{y}-\mathrm{c}}{\mathrm{~m}}=\chi \text { dividing both sides by } \mathrm{m} \text { the term in front of } \chi
\end{aligned}
$$

## Example 7

$$
a=3 b+6
$$

Make $b$ the subject of the formula
$a-6=3 b$ taking 6 from both sides

$$
\frac{a-6}{3}=b \text { dividing both sides by } 3 \text {, the term in front of } b
$$

## Example 8

$$
\mathrm{p}=\mathrm{hc}(\mathrm{c}+\mathrm{n})
$$

Make n the subject of the formula. First MULTIPLY out the brackets.

$$
\mathrm{p}=\mathrm{hc} \mathrm{c}^{2}+\mathrm{hcn}
$$

Now take hc² from both sides to give

$$
\mathrm{p}-\mathrm{hc} \mathrm{c}^{2}=\mathrm{hcn}
$$

Now divide both sides by hc to give,

$$
\frac{\mathrm{P}-\mathrm{hc}{ }^{2}}{\mathrm{hc}}=\mathrm{n}
$$

This is the answer. NB. You CANNOT CANCEL. Please see the pack on Algebraic Fractions if you need to refresh your memory.

## Example 9

$$
X \quad \frac{3(A-B)}{Y}=
$$

Make B the subject of the formula. First MULTIPLY both sides by Y.

$$
Y=3(A-B)
$$

Now multiply out the brackets, to give

$$
Y=3 A-3 B
$$

Put the term containing B on the opposite side of the equals sign, and everything else on the other side.

$$
3 B=3 A-Y
$$

Now divide both sides by 3

$$
B=\frac{3 A-X Y}{3}
$$

This is the answer. Do not forget - You CANNOT CANCEL here. WHY?

## Example 10

$$
\frac{w}{a}+\frac{y}{b}=1
$$

Make a the subject of the formula. MULTIPLY both sides by the LCM, which is ab in this formula.

$$
a b \times \frac{w}{a}+a b \times \frac{y}{b}=a b \times 1
$$

Cancel to give wb + ya = ab
Now subtract ya from both sides

$$
\mathrm{wb}=\mathrm{ab}-\mathrm{ya}
$$

Take out a as a common factor

$$
w b=a(b-y)
$$

Now divide both sides by (b-y)

$$
\frac{B w}{b-y}=a
$$

## Example 11

$$
V=\frac{2 P}{P-r}
$$

Make P the subject of the formula. MULTIPLY both sides by ( $\mathrm{P}-\mathrm{r}$ ), to give,

$$
V(P-r)=\frac{2 P(P-r)}{P-r}
$$

Cancel (p-r)

$$
V(P-r)=2 P
$$

Multiply out the bracket

$$
V P-V r=2 P
$$

Collect all the terms containing $P$ on one side.
Take out $P$ as a common factor

$$
P(V-2)=V r
$$

Divide both sides by (V-2)

$$
P=\frac{V r}{V-2}
$$

## Example 12

Make g the subject of the formula.

$$
\mathrm{t}=2 \pi \sqrt{\frac{1}{g}}
$$

SQUARE ALL the terms to remove the square root sign.

$$
\mathrm{t}^{2}=4 \pi^{2} \underline{1}
$$

g
NOTE:- when a square root is squared, the root sign is removed.
Multiply both sides by g

$$
\mathrm{gt}^{2}=4 \pi^{2}
$$

Divide both sides by $\mathrm{t}^{2}$

$$
g=\frac{4 \pi^{2}}{t^{2}}
$$

## Example 13

$$
\chi=\mathrm{a} \sqrt{P}-\mathrm{t}
$$

Make $P$ the subject of the formula. $P$ must be placed on one side, whilst the other terms are on the other side.

$$
\chi+\mathrm{t}=\mathrm{a} \sqrt{P}
$$

Now square both sides, to balance out the effect of the square root.

$$
(\chi+t)^{2}=a^{2} P
$$

Divide both sides by $\mathrm{a}^{2}$

$$
(\chi+t)^{2}=P
$$

Now try these. If you are in any doubt, please ask!

## Original Make this the subject

1. $\quad \mathrm{PV}=\mathrm{T}$

V
2. $2 \mathrm{~A}=\mathrm{PQ}$

P
3. $\mathrm{V} 2=4 \mathrm{gh}$
h
4. $\quad \mathrm{a}=\frac{\mathrm{p}}{\mathrm{q}}$
p
5. $\quad a=\frac{p}{q}$
q
6. $T=\frac{V A T}{500}$

A
7. $v=u+a t$
a
8. $\mathrm{H}=\mathrm{S}-\mathrm{dl}$ I
9. $\mathrm{S}=\mathrm{h}(\chi-\mathrm{t}) \mathrm{h}$
10. $T=\frac{2(S+P)}{Z} \quad S$
11. $\frac{a}{\chi}+\frac{b}{y}=1$

Y
12. $\mathrm{V}=\sqrt{g h}$
h
13. $\mathrm{V}=\mathrm{k} \sqrt{d}$
d
14. $\mathrm{t}=3 \mathrm{~h} \sqrt{\frac{\chi}{y}}$
$\chi$
15. $\mathrm{S}=\frac{3 \mathrm{P}}{\mathrm{P}+\chi}$

P

It is possible to obtain answers, which are DIFFERENT from those we have given you, if you have tried to put the subject on the other side to our answers. There is nothing wrong. If you need to check, please see the Learning Development Maths tutor.

## Learning

## ANSWERS

1. $V=\frac{T}{P}$
2. $P=\frac{2 A}{Q}$
3. $h=\frac{V^{2}}{4 g}$
4. $p=a q$
5. $q=\frac{P}{a}$
6. $A=\frac{500}{V}$
7. $a=\frac{v-u}{t}$
8. $I=\frac{S-H}{d}$
9. $h=\frac{S}{(\chi-t)}$
10. $S=\frac{Z T-2 P}{2}$
11. $y=\frac{b \chi}{\chi-a}$
12. $h=\frac{v^{2}}{g}$
13. $d=\frac{v^{2}}{k^{2}}$
14. $\chi=\frac{t^{2} Y}{9 h^{2}}$
15. $P=\frac{S \chi}{3-S}$
